
2026 BxMO Team Selection Test

Instructions. This test consists of **15 problems**. The answer to each problem is a non-negative integer from **000 to 999** inclusive. Time allowed: **3 hours**. No calculators, books, notes, or electronic devices. Each correct answer scores 1 point; no penalty for incorrect or blank answers.

Problem 1. If $x + \frac{1}{x} = 3$, find $x^6 + x^4 + x^2 + \frac{1}{x^2} + \frac{1}{x^4} + \frac{1}{x^6}$.

Problem 2. How many positive divisors of $12!$ are divisible by 60?

Problem 3. Find the number of ordered pairs (a, b) of positive integers with $a, b \leq 20$ such that $ab(a + b)$ is not divisible by 3.

Problem 4. Let $P(x)$ be a polynomial of degree 4 with leading coefficient 1 such that $P(1) = 1$, $P(2) = 3$, $P(3) = 5$, and $P(4) = 7$. Find $P(5)$.

Problem 5. In $\triangle ABC$, $AB = 13$, $BC = 14$, and $CA = 15$. Let I be the incenter of the triangle. The line passing through I parallel to BC intersects AB and AC at D and E , respectively. Find the perimeter of $\triangle ADE$.

Problem 6. How many integers from 1 to 999 have digit sum 12?

Problem 7. Find the sum of all positive integers x such that there exists a positive integer y satisfying $x \leq y$ and $\frac{1}{x} + \frac{1}{y} = \frac{1}{15}$.

Problem 8. In $\triangle ABC$, points D and E are on sides BC and CA , respectively, such that $BD : DC = 2 : 3$ and $CE : EA = 3 : 4$. Let P be the intersection of AD and BE . If the ratio of the area of $\triangle ABP$ to the area of $\triangle ABC$ is expressed as a fraction $\frac{m}{n}$ in lowest terms, find $m + n$.

Problem 9. Let S be the set of all subsets of $\{1, 2, \dots, 10\}$. For each $A \in S$, let $P(A)$ be the product of the elements of A , with the convention that the product of the empty set is 1. Find the remainder when $\sum_{A \in S} P(A)$ is divided by 1000.

Problem 10. An ant stands at the vertex $(0, 0, 0)$ of a cube whose vertices are the points (x, y, z) with $x, y, z \in \{0, 1\}$. At each step, the ant moves to an adjacent vertex by changing exactly one coordinate. How many walks of exactly 7 steps end at the vertex $(1, 1, 1)$?

Problem 11. How many triangles can be formed using 3 distinct vertices of a regular 21-gon such that the center of the 21-gon lies strictly inside the triangle?

Problem 12. Two circles ω_1 and ω_2 of radii 5 and 12, respectively, are externally tangent at T . Their common external tangents meet at X . Let M be the intersection of one of the common external tangents with the common internal tangent. If $XM = \frac{a\sqrt{b}}{c}$, where a and c are coprime positive integers and b is square-free, find $a + b + c$.

Problem 13. Let $a_1 = 2$, and for each integer $n \geq 1$, define $a_{n+1} = \frac{a_n}{\sqrt{1 + a_n^2}}$. Find $\sum_{k=1}^{100} \left\lfloor \frac{1}{a_k} \right\rfloor$.

Problem 14. An acute triangle ABC is inscribed in a circle of radius 25, and its orthocenter H satisfies $AH = 14$. Find the largest possible area of $\triangle ABC$.

Problem 15. Let $f : \mathbb{R} \setminus \{0, 1\} \rightarrow \mathbb{R}$ be a function satisfying

$$f(x) + f\left(\frac{1}{1-x}\right) = \frac{x^2 + 1}{2x}$$

for all $x \neq 0, 1$. Find the remainder when $\lfloor f(2026) \rfloor$ is divided by 1000.

Good luck!