
IMO 2026 Team Selection Test

Instructions. This test consists of **3 problems**. Time allowed: **4 hours**. Each problem is worth 7 points. Please write your solutions clearly, providing full mathematical justification. No calculators, books, notes, or electronic devices are permitted.

Problem 1. Let ABC be a triangle. Let X be a variable point on $[BA)$, the ray from B through A , such that $BX < BC$. Let Y be the unique point on $[CA)$, the ray from C through A , satisfying $BX + CY = BC$. Prove that, as X varies with $X, Y \neq A$, the circumcircle of triangle AXY passes through a fixed point other than A .

Problem 2. Some cells of a 2026×2026 grid of unit squares are initially colored red or blue, while the remaining cells are uncolored. The grid then evolves in steps: at each step, simultaneously, every uncolored cell that shares an edge with at least two red cells becomes red, and every uncolored cell that shares an edge with at least two blue cells becomes blue. If a cell qualifies for both colors simultaneously, either color may be chosen. Once colored, a cell never changes color.

Suppose that eventually every cell of the grid is colored, and that in the final configuration, all cells in the top row are red and all cells in the bottom row are blue. Determine the smallest possible total number of initially colored cells.

Problem 3. Let $f : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$ be a function such that for all positive integers a and b ,

$$f(\gcd(a, b)) \mid \gcd(f(a), f(b)),$$

and for all positive integers n ,

$$f^{(2n+2)}(n) = n.$$

Find the smallest possible value of $f(2025)$.

(Note: $f^{(k)}(x)$ denotes the function f applied k times to x .)

Good luck!